

# UNIVERSITY OF MUMBAI



**Syllabus for the S.Y.B.Sc.  
Program: B.Sc.  
Course : Mathematics**

(Credit Based Semester and Grading System with  
effect from the academic year 2012–2013)

**Revised syllabus in Mathematics**  
**As per credit based system**  
**Second Year B.A./B.Sc. 2012-13**

Name of the Programme	Duration	Semester	Subject
B.Sc. in Mathematics	Six semesters	III	Mathematics (courses: USMT301,USMT302,USMT303)
B.A. in Mathematics	Six semesters	III	Mathematics (courses: UAMT301, UAMT302)
Course Code	Title	Credits	
USMT301,UAMT301	Calculus and Analysis I	3 for USMT301,USMT302,USMT303,UAMT301,UAMT302	

**Teaching Pattern**

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One tutorial per week per batch per course. (The batches to be formed as prescribed by the University).
3. One assignment per week.

**Unit 1. Real Numbers (15 Lectures)**

- (a) (i) Statements of algebraic and order properties of  $\mathbb{R}$ .  
(ii) Elementary consequences of these properties including the A.M. - G.M. inequality, Cauchy-Schwarz inequality, and Bernoulli inequality (without proof).
- (b) (i) Review of absolute value and neighbourhood of a real number.  
(ii) Hausdorff property.
- (c) Supremum (lub) and infimum (glb) of a subset of  $\mathbb{R}$ , lub axiom of  $\mathbb{R}$ , Consequences of lub axiom of  $\mathbb{R}$  including
  - (i) Archimedian property.
  - (ii) Density of rational numbers.
  - (iii) Existence of  $n^{\text{th}}$  root of a positive real number (in particular square root).
  - (iv) Decimal representation of a real number.
- (d) (i) Nested Interval Theorem.

- (ii) Open sets in  $\mathbb{R}$  and closed sets as complements of open sets.
  - (iii) Limit points of a subset of  $\mathbb{R}$ , examples, characterisation of a closed set as a set containing all its limit points.
- (e) Open cover of a subset of  $\mathbb{R}$ , Compact subset of  $\mathbb{R}$ , Definition and examples. A closed and bounded interval  $[a, b]$  is compact.

**Reference for Unit 1:** *Chapter II, Sections 1, 2, 4, 5, 6 and Chapter X, Sections 1, 2 of Introduction to Real Analysis*, ROBERT G. BARTLE and DONALD R. SHERBET, *Springer Verlag*.

## Unit 2. Sequences, Limits and Continuity (15 Lectures)

- (a) Sequence of real numbers, Definition and examples. Sum, difference, product, quotient and scalar multiple of sequences.
- (b) Limit of a sequence, Convergent and divergent sequences, Uniqueness of limit of a convergent sequence, Algebra of convergent sequences, Sandwich Theorem of sequences. Limits of standard sequences such as

$$\left\{ \frac{1}{n^\alpha} \right\} \alpha > 0, \{a^n\} |a| < 1, \{n^{1/n}\}, \{a^{1/n}\} a > 0, \left\{ \frac{1}{n!} \right\}, \left\{ \frac{a^n}{n!} \right\} a \in \mathbb{R}$$

Examples of divergent sequences.

- (c) (i) Bounded sequences, A convergent sequence is bounded.
- (ii) Monotone sequences, Convergence of bounded monotone sequences, The number  $e$  as a limit of a sequence, Calculation of square root of a positive real number.
- (d) (i) Subsequences.
- (ii) Limit inferior and limit superior of a sequence.
- (iii) Bolzano-Weierstrass Theorem of sequences.
- (iv) Sequential characterisation of limit points of a set.
- (e) Cauchy sequences, Cauchy completeness of  $\mathbb{R}$ .
- (f) Limit of a real valued function at a point
  - (i) Review of the  $\varepsilon - \delta$  definition of limit of functions at a point, uniqueness of limits of a function at a point whenever it exists.
  - (ii) Sequential characterization for limits of functions at a point, Theorems of limits (Limits of sum, difference, product, quotient, scalar multiple and sandwich theorem).
  - (iii) Continuity of function at a point,  $\varepsilon - \delta$  definition, sequential criterion, Theorems about continuity of sum, difference, product, quotient and scalar multiple of functions at a point in the domain using  $\varepsilon - \delta$  definition or sequential criterion. Continuity of composite functions. Examples of limits and continuity of a function at a point using sequential criterion.

(iv) A continuous function on closed and bounded interval is bounded and attains bounds.

**Reference for Unit 2:** *Chapter III, Sections 1, 2, 3, 4, 5, Chapter IV, Sections 1, 2 and Chapter V, Sections 1, 2, 3 of Introduction to Real Analysis*, ROBERT G. BARTLE and DONALD R. SHERBET, *Springer Verlag*.

### Unit 3. Infinite Series (15 Lectures)

- (a) Infinite series of real numbers, The sequence of partial terms of an infinite series, convergence and divergence of series, sum, difference and multiple of convergent series are again convergent.
- (b) Cauchy criterion of convergence of series. Absolute convergence of a series, Geometric series.
- (c) Alternating series, Leibnitz' Theorem, Conditional convergence, An absolutely convergent series is conditionally convergent, but the converse is not true.
- (d) Rearrangement of series (without proof), Cauchy condensation test (statement only), application to convergence of  $p$  - series  $\sum \frac{1}{n^p}$  ( $p > 1$ ). Divergence of Harmonic series  $\sum \frac{1}{n}$ .
- (e) Tests for absolute convergence, Comparison test, Ratio test, Root test including proofs.
- (f) Power series, Radius of convergence of power series, The exponential, sine and cosine series.
- (g) Fourier series, Computing Fourier Coefficients of simple functions such as  $x$ ,  $x^2$ ,  $|x|$ , piecewise continuous functions on  $[-\pi, \pi]$ .

**The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise. Reference for Unit 3:** *Chapter IX, Sections 1, 2, 3, 4 and Chapter VIII, Sections*

*3, 4 of Introduction to Real Analysis*, ROBERT G. BARTLE and DONALD R. SHERBET, *Springer Verlag*.

### Recommended Books

1. ROBERT G. BARTLE and DONALD R. SHERBET : **Introduction to Real Analysis**, *Springer Verlag*.
2. R. COURANT and F. JOHN : **Introduction to Calculus and Analysis Vol I**, Reprint of First Edition, *Springer Verlag*, New York 1999.
3. R. R. GOLDBERG: **Methods of Real Analysis**, *Oxford and IBH Publication Company*, New Delhi.
4. T. APOSTOL: **Calculus Vol I**, Second Edition, *John Wiley*.
5. M. H. PROTTER: **Basic elements of Real Analysis**, *Springer Verlag*, New York 1998.

### Additional Reference Books

1. HOWARD ANTON, *Calculus - A new Horizon*, Sixth Edition, John Wiley and Sons Inc, 1999.
2. JAMES STEWART, *Calculus*, Third Edition, Brooks/cole Publishing Company, 1994.

### Suggested topics for Tutorials/Assignments

- (1) Properties of real numbers and Hausdorff property.
- (2) Bounded sets, finding l.u.b. and g.l.b. of sets.
- (3) Archmedian Property and Density Theorem.
- (4) Nested Interval and decimal representations.
- (5) Finding limit points of given sets.
- (6) Compact sets.
- (7) (i) Find limits of sequences using definition.  
(ii) Monotone sequences.
- (8) Subsequences, finding limit inferior and limit superior of given sequences.
- (9) Cauchy sequences.
- (10) Limits and continuity using sequential criterion.
- (11) Convergence of series. Comparison test.
- (12) Convergence of series: Root test, Ratio test.
- (13) Radius of convergence of a power series.
- (14) Fourier Series.

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B.A. in Mathematics	Six semesters	III	Mathematics (courses: UAMT301, UAMT302)
Course Code	Title	Credits	
USMT302, UAMT302	Linear Algebra I	3 for USMT301, USMT302, USMT303, UAMT301, UAMT302	

### Teaching Pattern

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One tutorial per week per batch per course. (The batches to be formed as prescribed by the University).

3. One assignment per week.

### Unit 1. Systems of linear equations and matrices (15 Lectures)

- (a) Systems of homogeneous and non-homogeneous linear equations.
  - (i) The solutions of systems of  $m$  homogeneous linear equations in  $n$  unknowns by elimination and their geometric interpretation for  $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$ .
  - (ii) Definition of  $n$ -tuples of real numbers, sum of two  $n$ -tuples and scalar multiple of  $n$ -tuple.
  - (iii) The existence of non-trivial solution of such a system for  $m < n$ . The sum of two solutions and a scalar multiple of a solution of such a system is again a solution of the system.
- (b)
  - (i) Matrices over  $\mathbb{R}$ , The matrix representation of systems of homogeneous and non-homogeneous linear equations.
  - (ii) Addition, scalar multiplication and multiplication of matrices, Transpose of a matrix.
  - (iii) The types of matrices: zero matrix, identity matrix, symmetric and skew symmetric matrices, upper and lower triangular matrix.
  - (iv) Transpose of product of matrices, Invertible matrices, Product of invertible matrices.
- (c)
  - (i) Elementary row operations on matrices, row echelon form of a matrix and Gaussian elimination method. Applications of Gauss elimination method to solve system of linear equations.
  - (ii) The matrix units, Row operations and Elementary matrices, Elementary matrices are invertible and an invertible matrix is a product of elementary matrices.

**Reference for Unit 1:** *Chapter II, Sections 1, 2, 3, 4, 5 of Introduction to Linear Algebra, SERGE LANG, Springer Verlag and Chapter 1, of Linear Algebra A Geometric Approach, S. KUMARESAN, Prentice-Hall of India Private Limited, New Delhi.*

### Unit 2. Vector spaces over $\mathbb{R}$ (15 Lectures)

- (a) Definition of a vector space over  $\mathbb{R}$ . Examples such as:
  - (i) Euclidean space  $\mathbb{R}^n$ .
  - (ii) The space  $\mathbb{R}^\infty$  of sequences over  $\mathbb{R}$ .
  - (iii) The space of  $m \times n$  matrices over  $\mathbb{R}$ .
  - (iv) The space of polynomials with real coefficients.
  - (v) The space of real valued functions on a non-empty set.
- (b) Subspaces - definition and examples including:
  - (i) Lines in  $\mathbb{R}^2$ , Lines and planes in  $\mathbb{R}^3$ .

- (ii) The solutions of homogeneous system of linear equations, hyperplane.
  - (iii) The space of convergent real sequences.
  - (iv) The spaces of symmetric, skew symmetric, upper triangular, lower triangular, diagonal matrices.
  - (v) The space of polynomials with real coefficients of degree  $\leq n$ .
  - (vi) The space of continuous real valued functions on  $[a, b]$ .
  - (vii) The space of continuously differentiable real valued functions on  $[a, b]$ .
- (c) (i) The sum and intersection of subspaces, direct sum of vector spaces.  
(ii) Linear combination of vectors, convex sets, linear span of a subset of a vector space.  
(iii) Linear dependence and independence of a set.
- (d) (The discussion of concepts mentioned below for finitely generated vector spaces only)  
Basis of a vector space, basis as a maximal linearly independent set and a minimal set of generators. Dimension of a vector space.
- (e) (i) Row space, Column space of an  $m \times n$  matrix over  $\mathbb{R}$  and row rank, column rank of a matrix  
(ii) Equivalence of row rank and column rank, Computing rank of a matrix by row reduction.

**Reference for Unit 2:** *Chapter III, Sections 1, 2, 3, 4, 5, 6 of Introduction to Linear Algebra*, SERGE LANG, *Springer Verlag* and *Chapter 2, of Linear Algebra A Geometric Approach*, S. KUMARESAN, *Prentice-Hall of India Private Limited*, New Delhi. .

### Unit 3. Inner Product Spaces (15 Lectures)

- (a) Dot product in  $\mathbb{R}^n$ , Definition of general inner product on a vector space over  $\mathbb{R}$ .  
Examples of inner product including the inner product  $\langle f, g \rangle = \int_{-\pi}^{\pi} f(t)g(t)dt$  on  $C[-\pi, \pi]$ , the space of continuous real valued functions on  $[-\pi, \pi]$ .
- (b) (i) Norm of a vector in an inner product space. Cauchy-Schwarz inequality, triangle inequality.  
(ii) Orthogonality of vectors, Pythagoras theorem and geometric applications in  $\mathbb{R}^2$ , projections on a line, The projection being the closest approximation.  
(iii) Orthogonal complements of a subspace, Orthogonal Complements in  $\mathbb{R}^2$  and  $\mathbb{R}^3$ .  
(iv) Orthogonal sets and orthonormal sets in an inner product space. Orthogonal and orthonormal bases. Gram-Schmidt orthogonalization process, simple examples in  $\mathbb{R}^3, \mathbb{R}^4$ .

**Reference for Unit 3:** *Chapter VI, Sections 1, 2 of Introduction to Linear Algebra*, SERGE LANG, *Springer Verlag* and *Chapter 5, of Linear Algebra A Geometric Approach*, S. KUMARESAN, *Prentice-Hall of India Private Limited*, New Delhi.

The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.

### Recommended Books

1. SERGE LANG: *Introduction to Linear Algebra*, Springer Verlag.
2. S. KUMARESAN: *Linear Algebra A geometric approach*, Prentice Hall of India Private Limited.

### Additional Reference Books

1. M. ARTIN: *Algebra*, Prentice Hall of India Private Limited.
2. K. HOFFMAN and R. KUNZE: *Linear Algebra*, Tata McGraw-Hill, New Delhi.
3. GILBERT STRANG: *Linear Algebra and its applications*, International Student Edition.
4. L. SMITH: *Linear Algebra*, Springer Verlag.
5. A. RAMACHANDRA RAO and P. BHIMA SANKARAN: *Linear Algebra*, Tata McGraw-Hill, New Delhi.
6. T. BANCHOFF and J. WERMER: *Linear Algebra through Geometry*, Springer Verlag Newyork, 1984.
7. SHELDON AXLER : *Linear Algebra done right*, Springer Verlag, Newyork.
8. KLAUS JANICH : *Linear Algebra*.
9. OTTO BRETCHER: *Linear Algebra with Applications*, Pearson Education.
10. GARETH WILLIAMS: *Linear Algebra with Applications*, Narosa Publication.

### Suggested topics for Tutorials/Assignments

- (1) Solving homogeneous system of  $m$  equations in  $n$  unknowns by elimination for  $(m, n) = (1, 2), (1, 3), (2, 2), (2, 3), (3, 3)$ .
- (2) Row echelon form, Solving system  $AX = B$  by Gauss elimination.
- (3) Subspaces: Determining whether a given subset of a vector space is a subspace.
- (4) Linear dependence and independence of subsets of a vector space.
- (5) Finding bases of vector spaces.
- (6) Rank of a matrix.
- (7) Gram-Schmidt method.
- (8) Orthogonal complements of subspaces of  $\mathbb{R}^3$  (lines and planes).



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B.A. in Mathematics	Six semester	III	Mathematics (courses: UAMT301, UAMT302)
Course Code	Title	Credits	
USMT303	Computational Mathematics I	3 for USMT301,USMT302,USMT303,UAMT301,	

### Teaching Pattern

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One practical per week per batch per course. (The batches to be formed as prescribed by the University).

### Unit 1. Algorithms (15 Lectures)

- (a) Definition of an algorithm, characteristics of an algorithm  
Selection and iterative constructs in pseudocode, simple examples such as
  - (i) Finding the number of positive and negative integers in a given set,
  - (ii) Finding absolute value of a real number,
  - (iii) Exchanging values of variables,
  - (iv) Sum of n given numbers.
- (b) Searching and sorting algorithms, including
  - (i) Finding maximum and/or minimum element in a finite sequence of integers.
  - (ii) The linear search and binary search algorithms of an integer x in a finite sequence of distinct integers.
  - (iii) Sorting of a finite sequence of integers in ascending order. Bubble sort and insertion sort.
- (c) Algorithms on integers:
  - (i) Computing quotient and remainder in division algorithm.
  - (ii) Converting decimal number to a binary number.
  - (iii) Modular exponent.

- (iv) Euclidean algorithm to find the g.c.d of two non-zero integers.
- (d) Algorithms on matrices:
  - (i) Addition and multiplication of matrices.
  - (ii) Transpose of a matrix.
  - (iii) Power of a matrix.
- (e) Complexity of algorithm: Big O notation, Growth of functions, Time complexity, Best case, Average case, Worst Case complexity.  
Using big O notation to express the best, average and worst case behaviour for sorting and searching algorithms.
- (f) Recursion, Examples including:
  - (i) Fibonacci sequence
  - (ii) Computing  $a^n$  for non-negative integer  $n$ .
  - (iii) Euclidean algorithm.
  - (iv) Searching algorithm
  - (v) Factorial of a non-negative integer.
 Comparison of recursive and iterative methods.

**Reference for Unit 1:** *Chapter 2, and Chapter 3, Sections 3.4, 3.5 of Discrete Mathematics and Its Applications*, KENNETH H. ROSEN, *McGraw Hill Edition*.

## Unit 2. Graphs (15 Lectures)

- (a) Introduction to graphs: Types of graphs: Simple graph, Multigraph, psuedograph, directed graph, directed multigraph. One example/graph model of each type to be discussed.
- (b) (i) Graph Terminology: Adjacent vertices, degree of a vertex, isolated vertex, pendant vertex in a undirected graph.  
(ii) The handshaking Theorem for an undirected graph. An undirected graph has an even number odd vertices.
- (c) Some special simple graphs: Complete graph, cycle, wheel in a graph, Bipartite graph, regular graph.
- (d) Representing graphs and graph isomorphism.
  - (i) Adjacency matrix of a simple graph.
  - (ii) Incidence matrix of an undirected graph.
  - (iii) Isomorphism of simple graphs.

- (e) Connectivity:
- (i) Paths, circuit (or cycle) in a graph.
  - (ii) Connected graphs, connected components in an undirected graph, A strongly connected directed graph, A weakly connected directed graph. A cut vertex.
  - (iii) Connecting paths between vertices.
  - (iv) Paths and isomorphisms.
  - (v) Euler paths and circuits, Hamilton paths and circuits.  
Dirac's Theorem, Ore's Theorem
  - (vi) Shortest path problem, The shortest path algorithm - Dijkstra's Algorithm.
- (f) Planar graphs, planar representation of graphs, Euler's formula. Kuratowski's Theorem (statement only).

**Reference for Unit 2:** *Chapter 8, Sections 8.1, 8.2, 8.3, 8.4, 8.5, 8.6, 8.7 of Discrete Mathematics and Its Applications, KENNETH H. ROSEN, McGraw Hill Edition.*

### Unit 3. Trees (15 Lectures)

- (a) (i) Trees: Definition and Examples.  
(ii) Forests, Rooted trees, subtrees, binary trees.  
(iii) Trees as models.  
(iv) Properties of Trees.
- (b) Application of Trees:
- (i) Binary Search Trees, Locating and adding items to a Binary Search Tree.
  - (ii) Decision Trees (simple examples).
  - (iii) Game Trees, Minimax strategy and the value of a vertex in a Game Tree. Examples of games such as Nim and Tic-tac-toe.
- (c) Tree Traversal, Traversal algorithm including preorder traversal, inorder traversal, postorder traversal using recursion.
- (d) (i) Spanning Tree, Depth-First Search and Breadth-First Search.  
(ii) Minimum Spanning Trees, Prim's Algorithm, Kruskal's Algorithm

**(The Proofs of the results in this unit are not required and may be omitted).**

**Reference for Unit 3:** *Chapter 9, Sections 9.1, 9.2, 9.3, 9.4, 9.5 of Discrete Mathematics and Its Applications, KENNETH H. ROSEN, McGraw Hill Edition.*

### Recommended Books

1. KENNETH H. ROSEN : **Discrete Mathematics and Its Applications**, *McGraw Hill Edition*.
2. BERNARD KOLMAN, ROBERT BUSBY, SHARON ROSS: **Discrete Mathematical Structures**, Prentice-Hall India.
3. NORMAN BIGGS: **Discrete Mathematics**, *Oxford*.
4. DOUGLAS B. WEST: **Introduction to graph Theory**, *Pearson*.

#### **Additional Reference Books**

1. FRANK HARARY, *Graph Theory*, Narosa Publication.
2. R.G. DROMEY, *How to Solve it by computers*, Prentice-Hall India.
3. GRAHAM, KNUTH and PATASHNIK: *Concrete Mathematics*, Pearson Education Asia Low Price Edition.
4. THOMAS H. CORMEN, CHARLES E. LEISENON and RONALD L. RIVEST: *Introduction to Algorithms*, Prentice Hall of India, New Delhi, 1998 Edition.

#### **Suggested topics for Practicals**

- (1) Linear and binary search, sorting.
- (2) Algorithms on integers and matrices.
- (3) Recursive algorithms.
- (4) (i) Drawing a graph, counting the degree of vertices and number of edges.  
(ii) Representing a given graph by an adjacency matrix and drawing a graph having given matrix as adjacency matrix.
- (5) Determining whether the given pairs of graphs are isomorphic.  
(Exhibiting an isomorphism between the isomorphic graphs or proving that none exists).
- (6) Determining whether the given graph is connected or not.  
Finding connected components of a graph.  
Finding strongly connected components of a graph.  
Finding cut vertices.
- (7) To determine whether the given graph is a tree. Construction of Binary Search Tree and applications to sorting and searching.
- (8) Spanning Trees. Finding Spanning Tree using Breadth First Search and/or Depth First Search.

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B.Sc. in Mathematics	Six semesters	<b>IV</b>	Mathematics (courses: USMT401,USMT402,USMT403)
B.A. in Mathematics	Six semesters	<b>IV</b>	Mathematics (courses: UAMT401, UAMT402)
Course Code	Title	Credits	
USMT401,UAMT401	Calculus and Analysis II	3 for USMT401,USMT402,USMT403,UAMT401,UAMT402	

### Teaching Pattern

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One tutorial per week per batch per course. (The batches to be formed as prescribed by the University).
3. One assignment per week.

#### Unit 1. Differential Equations (15 Lectures)

##### (a) First Order Differential Equations:

- (i) Review of separable differential equations, homogeneous and non-homogeneous differential equations.
- (ii) Exact differential equations and integrating factors. Rules for finding integrating factors of  $M(x, y)dx + N(x, y)dy = 0$  (without proof) when,
  - $\frac{\frac{\partial N}{\partial x} - \frac{\partial M}{\partial y}}{M} = h(y),$
  - $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{N} = g(x),$
  - $\frac{\frac{\partial M}{\partial y} - \frac{\partial N}{\partial x}}{yN - xM} = f(xy).$
- (iii) Linear differential equations and Bernoulli differential equations.
- (iv) Modeling with first order equations. Examples from Financial Mathematics, Chemistry, Environmental Science, Population growth and decay.

##### (b) Second order Linear Differential Equations:

- (i) The general second order linear differential equation. Existence and Uniqueness Theorem for the solutions of a second order initial value problem (statement only).
- (ii) Homogeneous and non-homogeneous second order linear differential equations:

- The space of solutions of the homogeneous equations as a vector space.
  - Wronskian and linear independence of the solutions.
  - The general solution of homogeneous differential equation. The use of known solutions to find the general solution of a homogeneous equations.
  - The general solution of a non-homogeneous second order equation, Complementary functions and particular integrals.
- (iii) The homogeneous equation with constant coefficients, auxiliary equation, the general solution corresponding to real and distinct roots, real and equal roots and complex roots of the auxiliary equation.
- (iv) Non-homogeneous equations: The method of undetermined coefficients. The method of variation of parameters.

**Reference for Unit 1:** *Chapter 2, Sections 7, 8, 9, 10 and Chapter 3, Sections 14, 15, 16, 17, 18, 19, 20 of Differential Equations with Applications and Historical Notes*, G.F. SIMMONS, *McGraw Hill*.

## Unit 2. Multiple integrals (15 Lectures)

Review of functions of two and three variables, partial derivatives and gradient of two or three variables.

- (a) Double integrals:
- (i) Definition of double integrals over rectangles.
  - (ii) Properties of double integrals.
  - (iii) Double integrals over bounded regions.
- (b) Fubini's Theorem (without proof), Double integrals as volumes.
- (c) Applications of Double integrals: Average value, Areas, Moments, Center of Mass.
- (d) Double integrals in polar form.
- (e) Triple integrals in Rectangular coordinates, Average, volumes.
- (f) Applications of Triple integrals: Mass, Moments, Parallel axis Theorem.
- (g) Triple integrals in Spherical and Cylindrical coordinates.

**Reference for Unit 2:** *Chapter 13, Sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6 of Calculus and Analytic Geometry*, G.B. THOMAS and R. L. FINNEY, *Ninth Edition, Addison-Wesley*, 1998.

## Unit 3. Integration of Vector Fields (15 Lectures)

- (a) Line Integrals, Definition, Evaluation for smooth curves. Mass and moments for coils, springs, thin rods.
- (b) Vector fields, Gradient fields, Work done by a force over a curve in space, Evaluation of work integrals.
- (c) Flow integrals and circulation around a curve.
- (d) Flux across a plane curve.
- (e) Path independence of the integral  $\int_A^B F \cdot dr$  in an open region,  $F$  being a vector field over the region and  $A, B$  points in the region. Conservative fields, potential function.
- (f) The Fundamental theorems of line integrals (without proof).
- (g) Flux density (divergence), Circulation density (curl) at a point.
- (h) Green's Theorem in plane (without proof), Evaluation of line integrals using Green's Theorem.

**Reference for Unit 3:** *Chapter 14 of 14.1, 14.2, 14.3, 14.4 Calculus and Analytic Geometry*, G.B. THOMAS and R. L. FINNEY, *Ninth Edition, Addison-Wesley, 1998.*

**The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.**

### Recommended Books

1. G.B. THOMAS and R. L. FINNEY, **Calculus and Analytic Geometry**, Ninth Edition, Addison-Wesley, 1998.
2. G.F. SIMMONS: **Differential Equations with Applications and Historical Notes**, *McGraw Hill.*
3. SUDHIR GOPPADE and BALMOHAN LIMAYE : **A course in Multivariable calculus and Analysis**, *Springer.*

### Additional Reference Books

1. E.A. CODDINGTON and R. CARLSON : *Linear Ordinary Differential Equations*, SIAM.
2. W.E. BOYCE and R.C. DIPRIMA: *Elementary Differential equations and Boundary value problems* , John Wiley and Sons 8th Edition.
3. A.H. SIDDIQI and P. MANCHANDA : *A First Course in Differential Equations with Applications*, Macmillan.

### Suggested topics for Tutorials/Assignments

- (1) Solving first order exact equations and non-exact equations using integrating factors.
- (2) Linear equations, Bernoulli equations, Euler's equations
- (3) Wronskian and linear independence of solutions
- (4) Second order linear homogeneous equations with constant coefficient.
- (5) Method of undetermined coefficients, Method of variation of parameters
- (6) Double integrals, sketching regions, evaluation.
- (7) Triple integrals.
- (8) Application of Double and Triple integrals.
- (9) Evaluation of line integrals using definition, calculation of mass and moments for coil etc.
- (10) (i) Evaluation of work integrals
  - (ii) Flow integrals and flux across a plane curve.
  - (iii) Conservative fields and potential functions.

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B.A. in Mathematics	Six semesters	<b>IV</b>	Mathematics (courses: UAMT401, UAMT402)
Course Code	Title	Credits	
USMT402, UAMT402	Linear Algebra II	3 for USMT401, USMT402, USMT403, UAMT401, UAMT402	

### Teaching Pattern

1. Three lectures per week per course (1 lecture/period is of 48 minutes duration).
2. One tutorial per week per batch per course. (The batches to be formed as prescribed by the University).
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#### Unit 1. Linear Transformations (15 Lectures)

- (a) Linear transformations - definition and properties, examples including:
- (i) Natural projection from  $\mathbb{R}^n$  to  $\mathbb{R}^m$  ( $n \geq m$ )
  - (ii) The map  $L_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$  defined by  $L_A(X) = AX$ , where  $A$  is an  $m \times n$  matrix over  $\mathbb{R}$
  - (iii) Rotations and reflections in  $\mathbb{R}^2$ , Stretching and Shearing in  $\mathbb{R}^2$ .



- (iv) Orthogonal projections in  $\mathbb{R}^n$ .
- (v) Functionals.

The linear transformation being completely determined by its values on basis.

- (b) (i) The sum and scalar multiple of linear transformations from  $U$  to  $V$  where  $U, V$  are finite dimensional vector spaces over  $\mathbb{R}$  is again a linear transformation.
- (ii) The space  $L(U, V)$  of linear transformations from  $U$  to  $V$ .
- (iii) The dual space  $V^*$  where  $V$  is finite dimensional real vector space.
- (c) (i) Kernel and image of a linear transformation.
- (ii) Rank-Nullity Theorem.
- (iii) The linear isomorphisms, inverse of a linear isomorphism.
- (iv) Composite of linear transformations.
- (d) (i) Representation of a linear transformation from  $U$  to  $V$ , where  $U$  and  $V$  are finite dimensional real vector spaces by matrices with respect to the given ordered bases of  $U$  and  $V$ . The relation between the matrices of linear transformation from  $U$  to  $U$  with respect to different bases of  $U$ .
- (ii) Matrix of sum of linear transformations and scalar multiple of a linear transformation.
- (iii) Matrices of composite linear transformation and inverse of a linear transformation.
- (e) Equivalence of rank of an  $m \times n$  matrix  $A$  and rank of the linear transformation  $L_A : \mathbb{R}^n \rightarrow \mathbb{R}^m$  ( $L_A(X) = AX$ ). The dimension of solution space of the system of linear equations  $AX = 0$  equals  $n - \text{rank } A$ .
- (f) The solutions of non-homogeneous systems of linear equations represented by  $AX = B$ .
  - (i) Existence of a solution when  $\text{rank}(A) = \text{rank}(A, B)$ .
  - (ii) The general solution of the system is the sum of a particular solution of the system and the solution of the associated homogeneous system.

**Reference for Unit 1:** *Chapter VIII, Sections 1, 2 of Introduction to Linear Algebra, SERGE LANG, Springer Verlag and Chapter 4, of Linear Algebra A Geometric Approach, S. KUMARESAN, Prentice-Hall of India Private Limited, New Delhi.*

## Unit 2. Determinants (15 Lectures)

- (a) Definition of determinant as an  $n$ -linear skew-symmetric function from

$$\mathbb{R}^n \times \mathbb{R}^n \times \dots \times \mathbb{R}^n \rightarrow \mathbb{R}$$

such that determinant of  $(E^1, E^2, \dots, E^n)$  is 1, where  $E^j$  denotes the  $j^{\text{th}}$  column of the  $n \times n$  identity matrix  $I_n$ . Determinant of a matrix as determinant of its column vectors (or row vectors).

- (b) (i) Existence and uniqueness of determinant function via permutations (without proof).
- (ii) Computation of determinant of  $2 \times 2$ ,  $3 \times 3$  matrices, diagonal matrices.
- (iii) Basic results on determinants such as  $\det(A^t) = \det(A)$ ,  $\det(AB) = \det(A)\det(B)$ .
- (iv) Laplace expansion of a determinant (statement only), Vandermonde determinant, determinant of upper triangular and lower triangular matrices.
- (c) (i) Linear dependence and independence of vectors in  $\mathbb{R}^n$  using determinants.
- (ii) The existence and uniqueness of the system  $AX = B$ , where  $A$  is an  $n \times n$  matrix with  $\det(A) \neq 0$ .
- (iii) Cofactors and minors, Adjoint of an  $n \times n$  matrix  $A$ . Basic results such as  $A \cdot \text{adj}(A) = \det(A) \cdot I_n$ . An  $n \times n$  real matrix  $A$  is invertible if and only if  $\det A \neq 0$ ;  $A^{-1} = \frac{1}{\det A}(\text{adj}A)$  for an invertible matrix  $A$ .
- (iv) Cramer's rule.
- (d) Determinant as area and volume.

**Reference for Unit 2:** *Chapter VI of Linear Algebra A geometric approach*, S. KUMARESAN, *Prentice Hall of India Private Limited*, 2001 and *Chapter VII Introduction to Linear Algebra*, SERGE LANG, *Springer Verlag*.

### Unit 3. Eigenvalues and eigenvectors (15 Lectures)

- (a) (i) Eigenvalues and eigenvectors of a linear transformation  $T : V \rightarrow V$ , where  $V$  is a finite dimensional real vector space, Eigenvalues and eigenvectors of linear transformations listed in unit 1[a] as examples.
- (ii) Eigenvalues of  $n \times n$  real matrices.
- (iii) The linear independence of eigenvectors corresponding to distinct eigenvalues of a linear transformation.
- (b) (i) The characteristic polynomial of an  $n \times n$  real matrix, characteristic roots.
- (ii) Similar matrices, characteristic polynomials of similar matrices.
- (c) The characteristic polynomial of a linear transformation  $T : V \rightarrow V$ , where  $V$  is a finite dimensional real vector space.

**Reference for Unit 3:** *Chapter VIII, Sections 1, 2 of Introduction to Linear Algebra*, SERGE LANG, *Springer Verlag* and *Chapter 7, of Linear Algebra A Geometric Approach*, S. KUMARESAN, *Prentice-Hall of India Private Limited*, New Delhi.

The proofs of the results mentioned in the syllabus to be covered unless indicated otherwise.

### Recommended Books

1. SERGE LANG: *Introduction to Linear Algebra*, Springer Verlag.
2. S. KUMARESAN: *Linear Algebra A geometric approach*, Prentice Hall of India Private Limited.

### **Additional Reference Books**

1. M. ARTIN: *Algebra*, Prentice Hall of India Private Limited.
2. K. HOFFMAN and R. KUNZE: *Linear Algebra*, Tata McGraw-Hill, New Delhi.
3. GILBERT STRANG: *Linear Algebra and its applications*, International Student Edition.
4. L. SMITH: *Linear Algebra*, Springer Verlag.
5. A. RAMACHANDRA RAO and P. BHIMA SANKARAN: *Linear Algebra*, Tata McGraw-Hill, New Delhi.
6. T. BANCHOFF and J. WERMER: *Linear Algebra through Geometry*, Springer Verlag Newyork, 1984.
7. SHELDON AXLER : *Linear Algebra done right*, Springer Verlag, Newyork.
8. KLAUS JANICH : *Linear Algebra*.
9. OTTO BRETCHER: *Linear Algebra with Applications*, Pearson Education.
10. GARETH WILLIAMS: *Linear Algebra with Applications*, Narosa Publication.

### **Suggested topics for Tutorials/Assignments**

- (1) Linear transformations.
- (2) Determining kernel and image of linear transformations.
- (3) Matrices of linear transformations.
- (4) Solutions of system of linear equations.
- (5) Determinants: Computing determinants by Laplace's expansion.
- (6) Applications of determinants: Cramer's rule.
- (7) Finding inverses of  $2 \times 2$ ,  $3 \times 3$  invertible matrices using adjoint.
- (8) Finding characteristic polynomial, eigenvalues of  $2 \times 2$  and  $3 \times 3$  matrices.
- (9) Finding characteristic polynomial, eigenvalues and eigenvectors of linear transformations listed in unit 1[a] as examples.

Name of the Programme	Duration	Semester	Subject
B.Sc.in Mathematics	Six Semesters	IV	Mathematics ( Courses: USMT401,USMT402,USMT403 )
B.A.in Mathematics	Six Semesters	IV	Mathematics ( Courses: UAMT401,UAMT402 )
Course Code	Title	Credits	
USMT 403	Computational Mathematics II	3 for USMT401,USMT402,USMT403,UAMT401,UAMT402	

### Teaching Pattern

1. Three lectures per week per course (1 lecture/Period is of 48 minutes duration.)
2. One Practical per week per batch per course (The batches to be formed as prescribed by the University.)

### Unit 1. Application of Integration (15 Lectures)

- (a) (i) Area between two curves.  
(ii) Volumes by slicing, volumes of solids of revolution.  
(iii) Lengths of plane curves.  
(iv) Areas of surfaces of revolution.
- (b) (i) Improper integrals of two types:  
(1) The limits of integration are infinite i.e.  $\int_a^\infty f(x)dx$ ,  $\int_{-\infty}^\infty f(x)dx$ .  
(2) Integrand being infinite at one of the end points or an interior point.
- (ii) Convergence of improper integrals, Tests of convergence and divergence. Direct comparison test and limit form of comparison test, Evaluation of convergent improper integrals.
- (iii) Applications  
(1) Finding area of an infinite region.  
(2) Volume of solids of revolution of infinite region about  $x - axis$  or  $y - axis$ .
- (c) Euler's Gamma function and Stirling formula.

**Reference for Unit 1 :** *Chapter 5, Sections 5.1, 5.2, 5.3, 5.4, 5.5, 5.6 and Chapter 7, Section 7.6 of Calculus and Analytic Geometry, G.B.THOMAS and R.L.FINNEY, Ninth Edition, Addison – Wesley,1998.*

## Unit 2. Numerical Methods (15 Lectures)

- (a) Roots of equations in one variable: Bisection method, Newton Raphson method, Secant method, Fixed point iteration method and to use it to find roots of equations. Convergence, limitations and algorithm for each of the above methods.
- (b) Newton Raphson method for a system of non-linear equations, Multiple roots by Newton Raphson method and polynomial deflation.
- (c) Roots of polynomial, Fundamental Theorem of Algebra (statement only), Descarte's rule of sign, Muller's method.
- (d)  $LU$  factorization of a matrix where  $L$  is a lower triangular matrix and  $U$  is an upper triangular matrix, Doolittle LU decomposition, Cholesky decomposition.

**Reference for Unit 2:** *Chapter 6, Sections 6.1, 6.2, 6.3, 6.5, 6.6, 6.8, 6.9, 6.10, 6.12, 6.13, 6.14, 6.16 and Chapter 7, Section 7.7 of Numerical Methods, E. BALAGURUSWAMY, TATA McGraw Hill.*

## Unit 3. Numerical Solution for Ordinary Differential Equations (15 Lectures)

- (a) Solution of Initial value problem of an ordinary first order differential equation:
  - (i) One step methods: Taylor series method, Picard's method, Euler's method, Heun's method, Polygon method, Runge-kutta method of 2<sup>nd</sup> order, 4<sup>th</sup> order.
  - (ii) Accuracy of one-step methods
- (b) Solution of Initial value problem of an ordinary first order differential equation:
  - (i) Multistep methods (Predictor - Corrector methods): Milne-Simpson method, Adams-Bashforth-Moulton method.
  - (ii) Accuracy of multistep methods

**Reference for Unit 3:** *Chapter 13, Sections 13.1, 13.2, 13.3, 13.4, 13.5, 13.6, 13.7, 13.8, 13.9 and of Numerical Methods, E. BALAGURUSWAMY, TATA McGraw Hill.*

### Recommended Books

1. G.B. THOMAS and R. L. FINNEY: *Calculus and Analytic Geometry, Ninth Edition, Addison-Wesley, 1998.*
2. E. BALAGURUSWAMY: **Numerical Methods, TATA McGraw Hill.**

### Additional Reference Books

1. GRAHAM, KNUTH and PATASHNIK: *Concrete Mathematics*, Pearson Education Asia Low Price Edition.
2. KENDALL ATKINSON : *An Introduction to Numerical Analysis*, Wiley Student Edition.
3. RICHARD BURDEN and DOUGLAS FAIRES: *Numerical Analysis*, Thomson Books/Cole.
4. THOMAS H. CORMEN, CHARLES E. LEISENON and RONALD L. RIVEST: *Introduction to Algorithms*, Prentice Hall of India, New Delhi, 1998 Edition.

### Suggested topics for Practicals

- (1) (a) Applications of integrals: Finding
  - (i) Area between two curves.
  - (ii) Volumes by slicing, volumes of solids of revolution.
  - (iii) Lengths of plane curves.
  - (iv) Areas of surfaces of revolution.
 (b) Improper Integrals.
- (2) Bisection Method, Newton Raphson method.
- (3) Secant method, fixed point iterative method to find root of equation, Muller's method.
- (4) LU decomposition (Doolittle method, Cholesky method).
- (5) Euler's method, Heun's method
- (6) Runge-Kutta method of 2<sup>nd</sup> order, 4<sup>th</sup> order.
- (7) Milne-Simpson method, Adams-Bashforth- Moulton method.

The scheme of examination for the revised courses in the subject of Mathematics at the Second Year B.A./B.Sc. will be as follows.

### Scheme of Examination

The performance of the learners shall be evaluated into two parts. The learners performance shall be assessed by Internal Assessment with 40% marks in the first part by conducting the Semester End Examinations with 60% marks in the second part. The allocation of marks for the Internal Assessment and Semester End Examinations are as shown below:-

#### (a) Internal assessment 40%

Courses with tutorials (Mathematics)

Sr. No.	Evaluation type	Marks
1	Two Assignments ( One Tutorial converted into assignment ) / Case studies / Project	20
2	One class Test ( Tutorial converted into test )	10
3	Active participation in routine class instructional deliveries / tutorials	05
4	Overall conduct as a responsible student , mannerism and articulation and exhibit of leadership qualities in organizing related academic actives	05

### Internal Assessment 40% (USMT 303, USMT 403)

Sr. No.	Evaluation type	Marks
1	Two Practical Tests each of 10 marks (One practical converted into test))	20
2	Journal Assessment	10
3	Active Participation	05
4	Overall Conduct	05

Illustration of format for Mathematics theory and tutorial:

Course	Credit Points	A1	A2 Tut.	Unit Test Tut.	Active Part. in Tut.	Tut. Book	Int. 40	Ext. 60	Total	Gr. Pt.	Letter grade
		10	10	10	5	5	16/40	24/60	100		
USMT101	3	7	8	7	4	4	30	30	60		
USMT102	3	6	7	9	4	4	30	28	58	5	B

Illustration of format for Mathematics Practical (USMT 303, USMT 403)

Course	Credit Points	A1	A2	Journal	Active Part.	Overall Conduct	Int. 40	Ext. 60	Total	Gr. Pt.	Letter Grade
		10	10	10	5	5	16/40	24/60	100		
USMT101	3	7	8	7	4	4	30	28	58	5	B

### (b) External Theory examination 60%

(i) Duration: - These examinations shall be of 2 Hours duration.

(ii) Theory Question Paper Pattern:-

1. There shall be four questions each of 15 marks.
2. On each unit there will be one question and the fourth one will be based on entire syllabus.
3. All questions shall be compulsory with internal choice within the questions.
4. Each question may be subdivided into sub-questions a, b, c, and the allocations of marks depend on the weightage of the topic.
5. Each question will be of 20 to 23 marks when marks of all the subquestions are added (including the options) in that question.

Questions		Marks
Q1	Based on Unit I	15
Q2	Based on Unit II	15
Q3	Based on Unit III	15
Q4	Based on Unit I,II,III	15
Total Marks		60

**Guidelines about conduct of Tutorials/Assignments:**

1. Tutorials

Conduct and Evaluation: The tutorials should be conducted in batches formed as per the University circular. The tutorial session should consist of discussion between the teacher and the students in which students should participate actively. Each tutorial session should be evaluated out of 10 marks on basis of participation of student and the average of total aggregate should be taken.

2. Assignments:

Conduct and Evaluation: The topic of the assignment and the questions should be given to the students at least one week in advance. The assignment should be such that it can be completed in 45 - 50 minutes by a student. The teachers may resolve the doubts of the students during the week, after which the students should submit the assignment. Each assignment should be evaluated out of 10 marks and the average of the total aggregate should be taken.

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